Anomalous low-field classical magnetoresistance in two dimensions

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The magnetoresistance of classical two-dimensional electrons scattered by randomly distributed impurities is investigated by numerical simulation. At low magnetic fields we find for the first time a negative magnetoresistance proportional to $|B|$. This unexpected behavior is shown to be due to a memory effect specific for backscattering events, which was not considered previously.

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The problem of magnetoresistance in metals and semiconductors has a long history, and a large amount of work, both experimental and theoretical, was devoted to this subject. Theoretically, the simplest situation is that of two-dimensional degenerate non-interacting electrons moving in a plane perpendicular to the magnetic field and scattered by static impurity potential (the 2D electron gas at low temperatures is also the situation for which most of the experiments were done). In this case the relevant electron energy is equal to the Fermi energy, and the conventional Boltzmann-Drude approach yields zero magnetoresistance, i.e. the longitudinal resistivity, ρ_{xx}, does not depend on magnetic field. Thus the origin of the observed magnetoresistance should be looked for beyond the Boltzmann theory.

Most of the research in this domain was focused on the low-field magnetoresistance arising from quantum interference effects (weak localization) [1]. Although the pioneering work of of Baskin *et al* [2] long ago demonstrated the importance of non-Boltzmann classical memory effects in magnetotransport (see also Refs. [3,4]), only in recent years it was fully recognized that there are not only quantum, but also purely classical reasons for magnetoresistance, [5,6,7,8,9], which may be either negative or positive depending on the type of impurity potential. Whatever is the case, the classical magnetoresistance appears as a consequence of memory effects which are beyond the Boltzmann approach.

For the case of strong short-range scatterers, with which we are concerned, there is a large negative magnetoresistance at $\beta \gtrsim 1$, [5,6,9] where $\beta = \omega_c \tau$, ω_c is the electron cyclotron frequency, and τ is the momentum relaxation time. This is related to the existence of "circling" electrons, [2,4] which never collide with the short-range scattering centers, the fraction of such electrons being

$$P = \exp(-2\pi R/\ell) = \exp(-2\pi/\beta), \qquad (1)$$

where $R = v/\omega_c$ is the cyclotron radius, v is the electron (Fermi) velocity, and $\ell = v\tau = (Nd)^{-1}$ is the electron mean free path. For the case of isotropic scattering, the corresponding magnetoresistance is given by the simple formula (valid with an accuracy better than 2%): [6,9]

$$\rho_{xx} = \rho_0(1 - P), \qquad (2)$$

where ρ_0 is the zero-field resistivity. This formula is in good agreement with numerical simulation [9] in the limit $d/\ell = Nd^2 \to 0$, where d is the effective diameter of the scattering centers and N is their concentration. Eq. (2), which is valid in this limit, predicts an exponentially small magnetoresistance at low fields, when $\beta << 1$. However for finite values of the parameter d/ℓ a parabolic negative magnetoresistance was found at $\beta << 1$. [9]

The purpose of this Letter is to report a numerical study of the low-field classical magnetoresistance of a 2D electron gas with short-range scattering centers. Unexpectedly, our numerical simulation reveals a new characteristic magnetic field defined by the relation $\beta = d/\ell << 1$. For $\beta << d/\ell$ the (negative) magnetoresistance is linear in B, while outside this interval, for $d/\ell << \beta << 1$ the dependence is parabolic. We explain the physics of this previously unknown phenomenon and we discuss its relevance to the experimental results for magnetotransport in random antidot arrays. [10,11]

In our simulation a point particle (electron) with a given absolute value of its velocity, v, is scattered by disks of diameter d randomly positioned on a plane inside a square box with periodic boundary conditions (the box size is a thousand disk diameters). Both the hard-disk (Lorentz) model, which exhibits anisotropic scattering, and a modified model with isotropic scattering are studied. To characterize the coverage, we introduce a dimensionless concentration of scatterers $c = \pi N d^2/4 << 1$.

The simulation procedure is identical with the one described in Ref. [9] : an initial electron position is chosen at random with an initial velocity along the x direction, the successive velocity directions are determined after each collision and the conductivity tensor is determined by calculating the integral of the velocity-velocity correlation function over a time of $t = 50\tau$. For each value of field and disk concentration we take an average over 10^2 independent disk configurations and, for each configuration, over 10^7 independent trials for the initial electron position (10 time more than in ref. [9]).

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Our main result is presented in Fig. 1, which shows a characteristic anomaly in magnetoresistance, $\Delta\rho = \rho_{xx}(B) - \rho_0$, around zero magnetic field followed by the anticipated parabolic dependence on β (hard disk model). The anomaly is more visible when $\Delta\rho$ is plotted against β^2, see inset. For still higher fields the curve follows Eq. 2, as described in Ref. [9]. Similar results were obtained for isotropic scattering and other values of c.
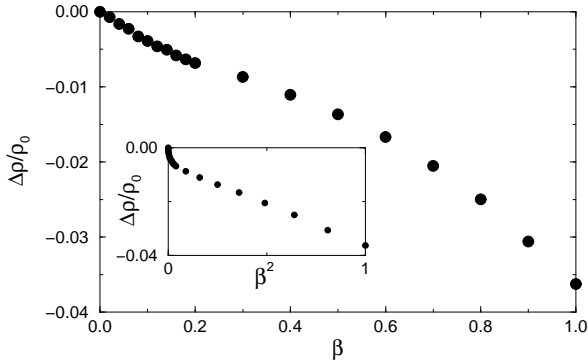

FIG. 1. Numerical results for the magnetoresistance $\Delta\rho/\rho_0$ as a function of $\beta = \omega_c\tau$ for $c = 0.1$ (hard disk model). Inset: same data plotted as a function of β^2. Here and below the error bars are within the symbol size.

To understand this result we need to discuss in some detail the nature of the classical non-Boltzmann corrections to the Drude resistivity. The Boltzmann approach is equivalent to a random redistribution of all scatterers after each collision. In reality there are memory effects of two types: (i) the particle may re-collide with the same impurity and (ii) the particle's trajectory may repeatedly pass through a space region which is free of impurities (the effect of circling electrons is of this type).

Consider first the case $B = 0$. The classical corrections to the Boltzmann equation due to re-collisions with the same scatterer (i) were studied long ago. [12] These processes are responsible for the $1/t^2$ tail in the velocity correlation function [13] and for the increase of the resistivity compared to its Drude value. The relative correction for small c is proportional to $c\ln(1/c)$ (or equivalently, to $(d/\ell)\ln(\ell/d)$), the logarithmic term being due to the simplest re-collision process $1 \rightarrow 2 \rightarrow 1$, while longer return loops give a smaller correction on the order of c.

As to the type (ii) processes, their role for $B = 0$ was not, to our knowledge, well understood so far. Consider a particle which, after going a distance $x \gg d$ without collisions, experiences backscattering at an angle $\phi = \pi$ and then returns to the initial point (Fig. 2a). [14] The probability of this round trip of length $2x$ is proportional to $\exp(-x/\ell)$, not to $\exp(-2x/\ell)$, as would suggest the conventional approach, since the existence of a free corridor of width d allowing the first part of the journey guarantees a collision-less return. This is not the case for scattering angles outside the interval on the order of d/x around the value $\phi = \pi$, when the probabilities for a free

path x before and after collision become independent and equal to $\exp(-x/\ell)$ each (Fig. 2b).

Since typically $x \sim \ell$, the probability of backscattering in the interval $\Delta\phi \sim d/\ell$ around $\phi = \pi$ is enhanced (the reversed velocity is conserved for a longer time), and this should lead to an additional increase of resistivity on the order of $d/\ell \sim c$, i.e. same order of magnitude as the contribution of return loops involving two or more intermediate scattering. One can say that the existence of a free corridor effectively enhances backscattering in the interval $\Delta\phi$, roughly by a factor of 2, thus increasing the transport cross-section by an amount $\sim d\Delta\phi$.

We attribute the low field anomaly in Fig. 1 to the influence of magnetic field on this effect. In the presence of even a small magnetic field the electron trajectories before and after collisions can not any more follow the same path (Fig. 2c). At high enough fields this kills the memory effect and, as a consequence, reduces the resistivity (Fig. 2d). Thus a negative magnetoresistance appears with a characteristic magnetic field defined by the relation $\beta = d/\ell \ll 1$.
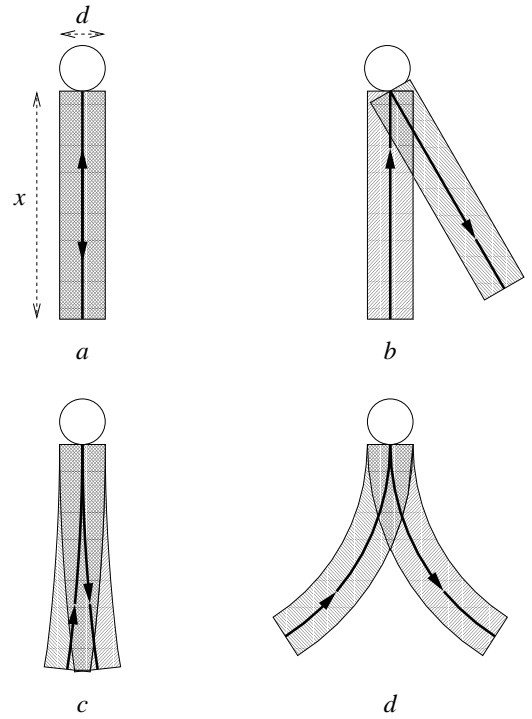

FIG. 2. Illustration of the memory effect in backscattering. a - Backscattering at angle π, $B = 0$. The particle returns through the same corridor. b - Scattering at an angle far from π, $B = 0$. The two corridors are different and the probabilities of free flights before and after collision are uncorrelated. c - Scattering at angle π in low magnetic field, $R \gg x^2/d$. The overlap of the two corridors diminishes. d - Same as c, but for higher field, $R \ll x^2/d$. The corridors cease to overlap.

We do not have a regular method for calculating analytically the magnetoresistance at low fields. However some qualitative conclusions may be drawn from simple

geometric considerations presented in Fig. 2.

Consider again backscattering by an angle exactly equal to π but in the presence of magnetic field (Fig. 2c). In order to have collision-less paths of length x before and after scattering, the centers of all disks should be outside the corridor of width d surrounding these paths. The probability of this is proportional to $P = \exp(-NS)$, where N is the disk concentration and S is the joint area of the two corridors. The overlapping region should not be counted twice.

While at $B = 0$ there is full overlap (Fig. 2a), $S = xd$ and $P = \exp(-x/\ell)$, in the presence of magnetic field the overlap diminishes and the relevant area increases. In the low-field limit, this increase can be easily calculated to be $\Delta S = x^3/(3R) \sim B$, so that P decreases *linearly* with B. This means that the (negative) magnetoresistance is linear in B for $R >> \ell^2/d$, or $\beta << d/\ell$. For higher fields, such that $\beta >> d/\ell$ the two corridors practically cease to overlap (Fig. 2d) and one has $S = 2xd$, $P = \exp(-2x/\ell)$. Similar considerations apply to backscattering in the interval $\Delta\phi \sim d/\ell$.

A similar contribution comes from the influence of magnetic field on the probability of the simplest re-collision process $1 \rightarrow 2 \rightarrow 1$, which necessarily involves backscattering in the same angular interval $\Delta\phi$. At $B = 0$ the memory effect increases the relative contribution of this process to the resistivity by an amount on the order of c, and again the curving of the trajectories in magnetic field will increase the total area S and thus reduce the probability of this process. In the low-field limit one finds again that the area increase, ΔS, is linear in B. These qualitative considerations lead us to the following conclusions concerning low-field classical magnetoresistance in two dimensions in the presence of appreciable backscattering.

1) A new characteristic magnetic field exists, at which the classical parameter $\beta = \omega_c\tau$ is small: $\beta_c = d/\ell \sim c << 1$.

2) The total drop of resistivity in the region $\beta \lesssim \beta_c$ is on the order of $d/\ell \sim c$.

3) At $\beta << \beta_c$ the resistivity ρ_{xx} is *linear* in magnetic field exhibiting the $|B|$ cusp observed in our simulation [15].

4) For $\beta_c << \beta << 1$ only quadratic in B corrections remain, which are on the order of $c\beta^2$. (It can be shown that these corrections come from the influence of the magnetic field on the contribution of return loops).

This means that at low fields, $\beta << 1$, the magnetoresistance, $\Delta\rho$, is described by the formula:

$$\Delta\rho/\rho_0 = -c(f(\beta/c) + A\beta^2), \qquad (3)$$

where A is a numerical constant and $f(\xi)$ is a function which behaves as $|\xi|$ for small values of its argument and saturates at some value on the order of 1 for $|\xi| >> 1$. A complete theory should give the explicit form of the function $f(\xi)$ and the numerical value of the constant A. For the hard disk model, we find $f(\xi) \approx 0.04|\xi|$ for $\xi \lesssim 2$, and $A \approx 0.3$.

Note, that Eq. 3 implies that at low field the slope of $\Delta\rho/\rho_0$ as a function of β is independent of c and that the derivative $(d/d\beta)(\Delta\rho/\rho_0)$ has a jump at $B = 0$ which is a numerical constant depending only on the relative efficiency of backscattering (≈ 0.08 for hard disks).

Since the conclusions above are not derived rigorously, we test them by plotting the relative magnetoresistance in units of c as a function of the dimensionless magnetic field β/c for several values of c. As follows from Eq. 3, in these units the magnetoresistance curves should coincide at low fields (where the function $f(\xi)$ is linear), but diverge for $\beta \gtrsim c$. This prediction is in excellent agreement with the simulation results, as presented in Fig. 3, which convinces us that our understanding is basically correct.
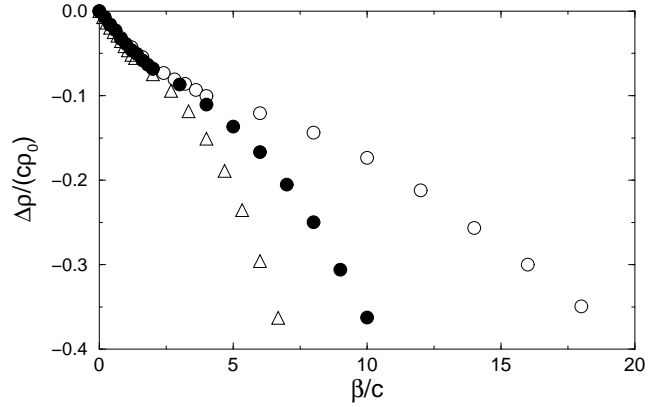

FIG. 3. Simulation results for several values of c plotted in units $\Delta\rho/(c\rho_0)$ versus β/c showing universal behavior at low fields as predicted by Eq. 3. Open circles, filled circles and open triangles correspond to $c = 0.05$, 0.1, and 0.15, respectively.

Finally, we compare our simulation with the experimental results for magnetoresistance of 2D electrons in a disordered array of antidots. [10,11] In these experiments the antidot diameter, d, is much greater than the Fermi wavelength, so that the classical approach is justified. Our numerical simulation exactly corresponds to the experimental situation at low temperatures.

Negative linear magnetoresistance at low fields, first observed in Ref. [10], quantitatively agrees with our results as seen in Fig. 4. The necessary data being given in Ref. [10], this comparison is done without any fitting parameters. (The main uncertainty is in the effective antidot diameter which is given in Ref. [10] as $d = 0.2 - 0.3\mu m$. We take $d = 0.25\mu m$. In order to re-plot the experimental curve we calculate $c \approx 0.1$ and $\beta = 12.5B(\text{T})$). It should be noted that while there is an excellent agreement for $\beta \lesssim 1$, strong deviations are found at $\beta > 1$. The discrepancy may be due to the fact that the antidot array used in these experiments was, in fact, a strongly distorted regular lattice with a substantial long-range order.
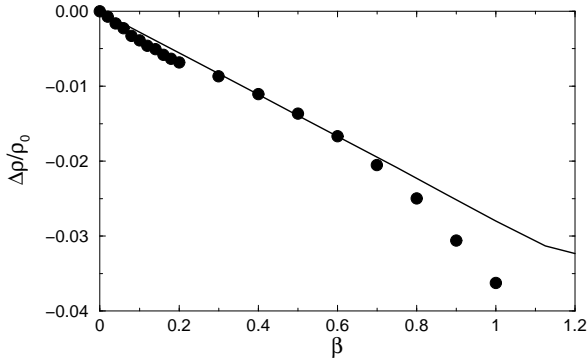
FIG. 4. Simulation results for magnetoresistance at $c = 0.1$ (filled circles) compared to the experimental curve of Ref.[10] (thick line).

In Ref. [11] the observed anomaly is attributed to the quantum (weak localization) effect. Direct comparison of our results with the experiment is difficult because of the way in which the data is presented, however we find, at least, a qualitative agreement both in the magnitude of the low-field anomaly and in the value of the characteristic magnetic field at T=1.2 K. This may be an indication that the experimental results are (at least partly) a manifestation of the classical phenomenon presented above. A strong temperature dependence of the low-field anomaly was observed in Ref. [11], which seems to support its interpretation as a result of weak localization. However, the observed shrinking of the low-field anomaly, as well as of the value ρ_0, with increasing temperature could be also due to the reduction of the memory effect by electron-electron and phonon scattering. The temperature dependence of the anomalous magnetoresistance needs further studies.

The artificial random antidot array is an interesting object, and a systematic experimental study of magnetoresistance in this system is highly desirable to separate the classical and quantum effects in magnetotransport.

A regular analytical calculation of the classical magnetoresistance at low fields remains a challenging task. It would also be important to understand the transition from classical to quantum behavior as the ratio of the Fermi wavelength to the scattering diameter increases.

Interestingly, a mechanism similar to the one considered here is responsible for the "opposition effect" [16] - the increase of the brightness of the Moon and other planets in a small angular interval around the position when the Sun, the planet, and the observer lie on the same straight line.

In summary, by numerical simulation we have found an anomalous low-field classical magnetoresistance of 2D electrons in the presence of strong short-range scattering. A new characteristic magnetic field is found, at which the classical parameter $\beta = \omega_c \tau$ is small: $\beta_c = d/\ell << 1$. For $\beta << \beta_c$ the negative magnetoresistance is proportional to $|B|$, and the slope of $\Delta\rho/\rho_0$ versus β is independent of the impurity concentration. We have shown that this phenomenon is due to a specific memory effect associated with backscattering.

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[14] Backscattering by an angle *exactly* equal to π would completely reverse the previous classical trajectory. What really matters, are scattering events in a small angular interval around π. If this interval is small enough, the electron may return to the previous scatterer (the process $1 \rightarrow 2 \rightarrow 1$) or even approximately reverse a part of its trajectory. Physically, the relevant angular interval is on the order of d/l.

[15] Strictly speaking, the magnetoresistance is an analytical function of B: a careful analysis shows that, in a very small region around zero field, where $\beta \lesssim c^2$, the dependence on B should be parabolic. This region is not accessible in our simulation.

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